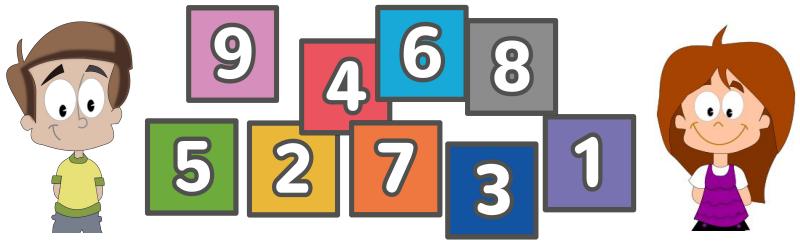
Magical Maths

Number Day is led by NSPCC

We have chosen some magical connected problems that have relatively easy entry points so that all students can begin, but also has scope for exploration and challenge for students at all levels. The problems will allow students to think, reason and make decisions – in other words, to work like mathematicians at a level and pace appropriate to them. We hope you enjoy exploring the activities in this handout. There is a video to watch as well: https://bit.ly/4oUkMka Let's start with the game Fifteen.

Game - Fifteen

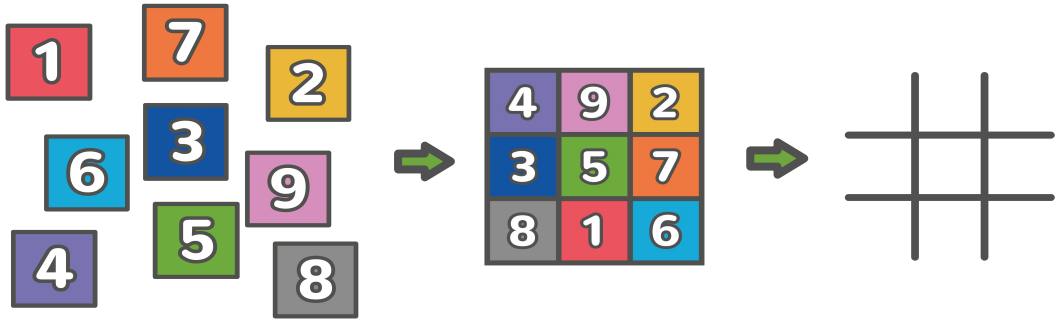


You will need: Two players and 1 - 9 digit cards or suitable alternative like dot cards

How to play: Take it in turns to remove cards from the pile. First person that has any three cards that sum to 15 wins.

Challenge: Can you find a winning strategy? Does it matter who goes first? Which digit card is it best to start with? What are the winning combinations? Can you block your opponent?

What does the game remind you of?



Some students are likely to pick up the similarities with tic-tac-toe (noughts and crosses):

- a winning set consists of three elements
- some moves block potential sets from your opponent
- sometimes a player wins by setting up two winning sets, only one of which could be blocked
- some games end in a tie

To make the link between our game and tic-tac-toe, we can use magic squares!

What's 'magic' about a magic square?

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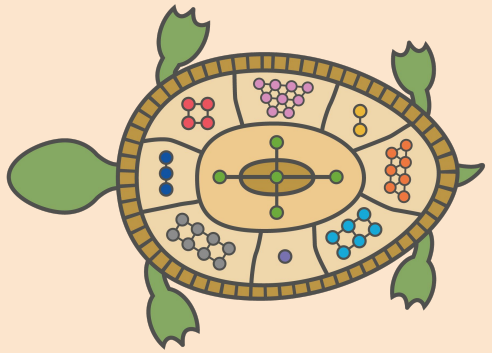
In the ancient Chinese legend of 'Lo Shu', there's a story about a huge flood that destroyed all the crops and land. People offered sacrifices to the river god so that one of the flooded rivers (the Lo river) would calm down. Every time the river flooded, a turtle would walk around the sacrifice. One day a child noticed a pattern on the shell of the turtle. It told people how many sacrifices they need to make for the god to accept. The pattern consisted of circular dots arranged in a 3-by-3 grid, where the number of dots in each column, row and diagonal summed to the same amount, fifteen. So, they realised they needed to make 15 sacrifices to keep the god happy.



4	9	2
3	5	7
8	1	6

To create a 3 x 3 magic square, you have to place the digits 1 – 9 into a 3 x 3 grid so the rows, columns and diagonals have the same total, called the '***magic constant'***.

Can you work out the magic constant?

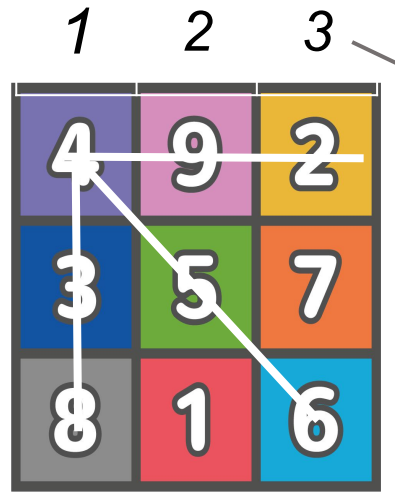
As it's a 3 x 3 grid we say it has an ***order of 3***. The magic number can be found by adding all the numbers in the square (we have 1 to 9 equals 45) and dividing by 3 (the order). 45 ÷ 3 = 15.

So, 15 is the magic constant for this magic square and we can begin to see how it relates to our Fifteen game. Normal magic squares always have the same magic constant.

What would the magic constant be for 4x4? 5x5? 6x6?

There is a formula for the normal magic square.

Have a go- does it work?



4 + 9 + 2 = 15
4 + 5 + 6 = 15
4 + 3 + 8 = 15

magic Constant

$$S = n \times \frac{n^2 + 1}{2}$$

the order

Is there more than one way to complete a magic square?

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There is magic to be found in the symmetry of how all the lines add up to the same amount.

4 9 2 3 5 7 8 1 6	8 3 4 1 5 9 6 7 2	6 1 8 7 5 3 2 9 4
Original	Rotation $\frac{1}{4}$	Rotation $\frac{1}{2}$
2 7 6 9 5 1 4 3 8	2 9 4 7 5 3 6 1 8	8 1 6 3 5 7 4 9 2
Rotation $\frac{3}{4}$	Reflection through vertical	Reflection through horizontal

There are six different symmetries of a 3 x 3 magic square with the same set of numbers. Look at how the magic square rotates and reflects.

There is more than one way to complete a magic square.

How many can you find?

Is the centre number in a 3 x 3 magic square always one third of the magic constant?

2	7	6
9	5	1
4	3	8

In a magic square all the rows, columns and diagonals add to the 'Magic Constant'.

How could you change the magic constant of this magic square?

The Challenge

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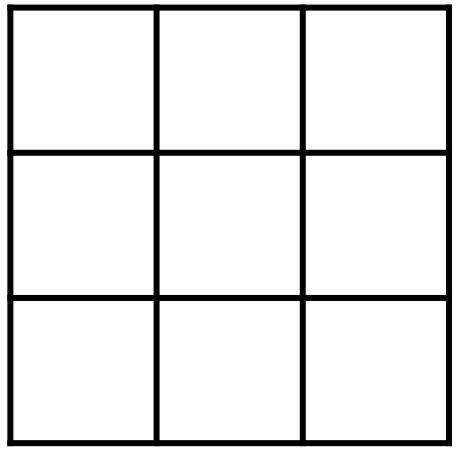
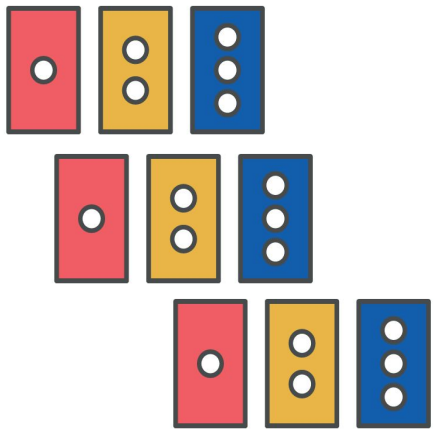
This is a different version of a magic square using just 1, 2, and 3.You can use number cards or dot cards. The numbers can be placed in a grid.

What is the magic constant? Which number will go in the centre?

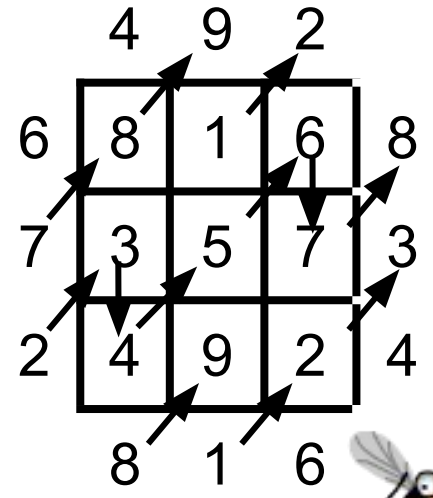




The **Siamese method, also known as de la Loubère method**, is a fascinating way to construct a magic square of odd order (e.g., 3x3, 5x5, 7x7, and so on). It's named after the French diplomat Simon de la Loubère, who popularised it in the 17th century.

Here's a brief outline of how it works, see if you can follow the path of the arrows:

- Start in the middle of the top row with **number 1**.
- **Move up and right** to place the next number.
- If the move takes you **out of bounds** (out of the grid):
  - Go to the bottom of the column if you move above the top.
  - Go to the left side of the row if you go beyond the right.
- If the space is already filled, **go down one row** instead and place the next number.



Have a go at completing the magic square using this method!

4 x 4 magic square and non normal magic square

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Albrecht Durer, a German painter, incorporated the magic square into his copperplate engraving. This particular magic square is famous. Even the year of the engraving is included 1514. **It is a Gnomon magic square.**

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

What is the magic constant?

What else do you notice about the Gnomon magic square?

How many ways can you make 34 using 4 numbers from 1 - 16?

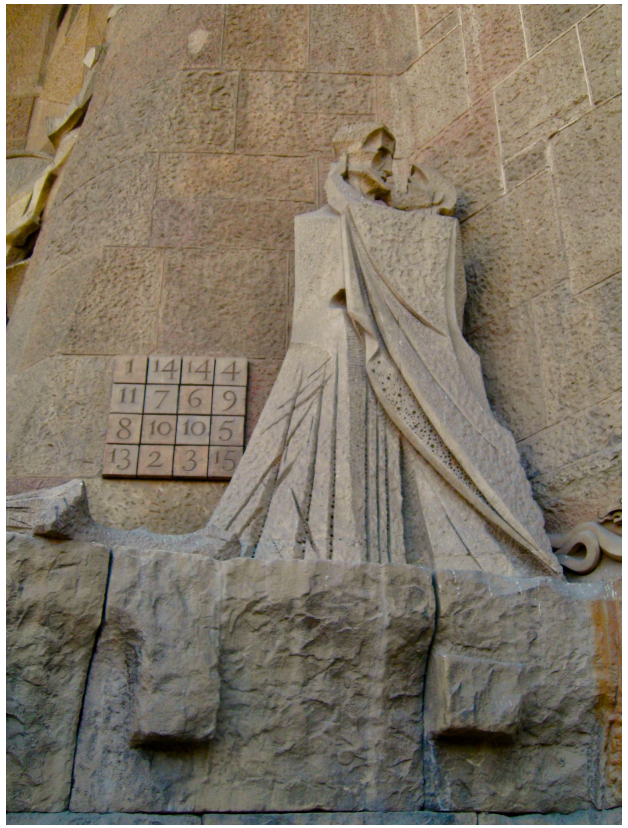
- The numbers used are 1 - 16
- The sum of 1 to 16 is 136
- The numbers in the 4 corners add up to 34
- The 4 centre numbers add up to 34
- The sum of the 4 quadrants is 34

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

This is a non-normal magic square because it repeats the numbers 10 and 14 and it doesn't use consecutive numbers.



Magic Square Appears in Passion Facade of the Sagrada Familia, Barcelona.

What is the magic constant of this magic square?

1	14	14	4
11	7	6	9
8	10	10	5
13	2	3	15

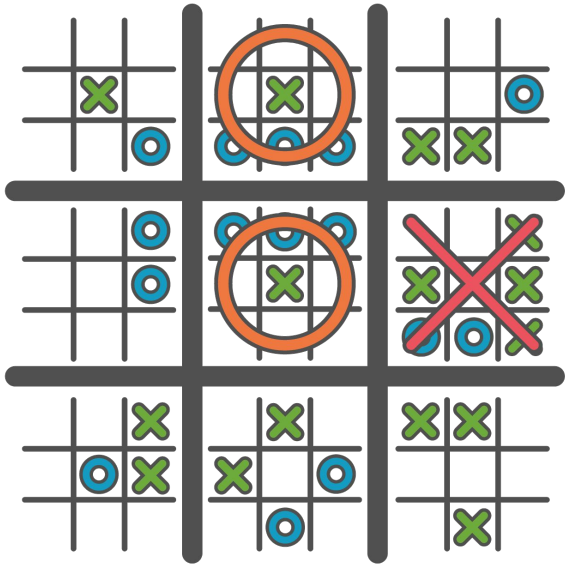
1	14	14	4
11	7	6	9
8	10	10	5
13	2	3	15

1	14	14	4
11	7	6	9
8	10	10	5
13	2	3	15

Look at the various ways the numbers can be summed to reach the magic constant.

Have a go at finding more within the square!

Further challenges...



Ultimate tic-tac-toe is a larger strategic game where the smaller tic-tac-toe games dictate the flow of the larger game. The player who can strategically control the smaller boards to win three in a row on the large board wins the overall game.

Watch the video to find out the rules!
https://bit.ly/4oUkMka

- Use **concrete resources** to fill the squares
- **Change** the numbers and the magic constant
- Try **different sizes** - 4 x 4, 5 x 5, 6 x 6 magic square
- **Gnomon square** - what are its properties?
- **Prime magic square** - Place numbers 1 - 9 so all the rows and columns add up to a prime number
- **Un-magic square** - Place numbers 1 - 9 so all the totals of each row, column and diagonal are different
- **Panmagic square** - what is this?
- Dotty six (Nrich)
- Traffic lights (Nrich)
- Play ultimate tic-tac-toe

You can hopefully see why we chose these rich problems. We started with a simple game of addition and ending up in the realm of tic-tac-toe, a game without numbers! We found a surprising connection between two very different looking ideas by navigating the mystery of magic squares. Maths is all about finding these hidden connections.